4–1 Multiplying Rational Numbers

What You'll Learn
You'll learn to multiply rational numbers.

Why It's Important
Health One way to determine safe backpack weights is to multiply rational numbers.
See Exercise 16.

Knowing how to multiply rational numbers is essential in simplifying algebraic expressions and solving equations. The multiplication of rational numbers can be modeled by repeated addition.

Find $4 \times (-0.2)$.

$$\begin{aligned} 4 \times (-0.2) &= -0.2 + (-0.2) + (-0.2) + (-0.2) \\ &= -0.8 \end{aligned}$$

This example illustrates the rule for multiplying two rational numbers.

Multiplying Two Rational Numbers	Words	Numbers
Different Signs	The product of two rational numbers with different signs is negative.	$1.2(-4.5) = -5.4$ $-1.2(4.5) = -5.4$
Same Sign	The product of two rational numbers with the same sign is positive.	$3(2.8) = 8.4$ $-3(-2.8) = 8.4$

Examples

Find each product.

1 $2.3(-4)$

$2.3(-4) = -9.2$ *The factors have different signs. The product is negative.*

Prerequisite Skills Review
Operations with Decimals, p. 684

2 $-3.5(-0.8)$

$-3.5(-0.8) = 2.8$ *The factors have the same sign. The product is positive.*

Your Turn

a. $-5(-1.3)$ b. $-2.4(7.5)$ c. $8 \cdot (-3.2)$

Real World

Example
Sports Link

3 **A skydiver jumps from 12,000 feet. Solve the equation $h = 12{,}000 + (0.5)(-32.1)(576)$ to find his height after he free-falls for 24 seconds.**

$$\begin{aligned} h &= 12{,}000 + (0.5)(-32.1)(576) \\ &= 12{,}000 + (-16.05)(576) && \textit{Multiply 0.5 and } -32.1. \\ &= 12{,}000 + (-9244.8) && \textit{Multiply } -16.05 \textit{ and } 576. \\ &= 2755.2 && \textit{Add } 12{,}000 \textit{ and } -9244.8. \end{aligned}$$

After 24 seconds, the skydiver's height is 2755.2 feet.

You can use grid paper to model the multiplication of fractions.

Hands-On Algebra
Models

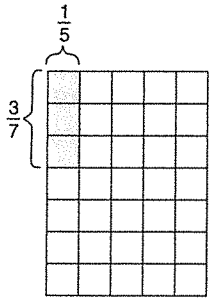
Materials: grid paper, colored pencils, straightedge

Multiply $\frac{3}{7}$ and $\frac{1}{5}$ using a model.

Step 1 Use a straightedge to draw a rectangle that has seven rows and five columns.

Step 2 Shade three of the seven rows yellow to represent $\frac{3}{7}$.

Step 3 Shade one column blue to represent $\frac{1}{5}$.



Try These

1. How many of the squares are shaded green?
2. What fraction of the total number of squares in the rectangle is shaded green?
3. The portion of squares that is shaded green represents the product of the fractions. What is the product of $\frac{3}{7}$ and $\frac{1}{5}$?
4. Use grid paper to model $\frac{3}{4}$ and $\frac{5}{8}$. Then use your model to find the product of $\frac{3}{4}$ and $\frac{5}{8}$.
5. What do you notice about the relationship between the numerators and denominators of $\frac{3}{4}$ and $\frac{5}{8}$ and their product?

The Hands-On Algebra activity suggests the following rule for multiplying fractions.

Multiplying Fractions

Words: To multiply fractions, multiply the numerators and multiply the denominators.

Symbols: $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$, where $b, d \neq 0$

Numbers: $\frac{1}{5} \cdot \frac{3}{4} = \frac{1 \cdot 3}{5 \cdot 4}$ or $\frac{3}{20}$

Examples

Find each product.

4 $-\frac{4}{5} \cdot \frac{1}{3}$

$-\frac{4}{5} \cdot \frac{1}{3} = -\frac{4 \cdot 1}{5 \cdot 3}$ *Multiply the numerators and multiply the denominators.*

$= -\frac{4}{15}$ *The factors have different signs. The product is negative.*

5 $6 \cdot \left(\frac{2}{5}\right)$

$6 \cdot \left(\frac{2}{5}\right) = \frac{6}{1} \cdot \frac{2}{5}$ *Rewrite 6 as an improper fraction.*

$= \frac{6 \cdot 2}{1 \cdot 5}$ *Multiply the numerators and multiply the denominators.*

$= \frac{12}{5}$ or $2\frac{2}{5}$ *The factors have the same sign. The product is positive.*

Prerequisite Skills Review
Simplifying Fractions, p. 685

6 $-\frac{1}{4} \cdot \left(-4\frac{1}{2}\right)$

$-\frac{1}{4} \cdot \left(-4\frac{1}{2}\right) = -\frac{1}{4} \cdot \left(-\frac{9}{2}\right)$ *Rewrite $-4\frac{1}{2}$ as an improper fraction.*

$= \left(\frac{1 \cdot 9}{4 \cdot 2}\right)$ *Multiply the numerators and multiply the denominators.*

$= \frac{9}{8}$ or $1\frac{1}{8}$ *The factors have the same sign. The product is positive.*

Your Turn

d. $\frac{3}{4}\left(\frac{5}{7}\right)$ e. $\frac{4}{5} \cdot (-3)$ f. $-\frac{1}{2}\left(-1\frac{3}{8}\right)$

Study these products.

$-1(5) = -5$ *-1 times 5 equals -5.*

$(-1)(-2.4) = 2.4$ *-1 times -2.4 equals 2.4.*

Notice that multiplying a number by -1 results in the opposite of the number. This suggests the **Multiplicative Property of -1.**

Multiplicative Property of -1

Words: The product of -1 and any number is the number's additive inverse.

Symbols: $-1 \cdot a = -a$ **Numbers:** $-1 \cdot \frac{2}{7} = -\frac{2}{7}$

You can use what you know about multiplying rational numbers to simplify algebraic expressions.

Example

7 **Simplify $-7.2t(5u)$.**

$$\begin{aligned} -7.2t(5u) &= (-7.2)(t)(5)(u) && -7.2t = (-7.2)(t);\ (5u) = (5)(u) \\ &= (-7.2)(5)(t)(u) && \textit{Commutative Property} \\ &= (-7.2 \cdot 5)(t \cdot u) && \textit{Associative Property} \\ &= -36tu && \textit{Simplify.} \end{aligned}$$

8 **Simplify $\frac{2}{5}m\left(\frac{3}{5}n\right)$.**

$$\begin{aligned} \frac{2}{5}m\left(\frac{3}{5}n\right) &= \left(\frac{2}{5}\right)(m)\left(\frac{3}{5}\right)(n) && \frac{2}{5}m = \left(\frac{2}{5}\right)(m);\ \left(\frac{3}{5}n\right) = \left(\frac{3}{5}\right)(n) \\ &= \left(\frac{2}{5}\right)\left(\frac{3}{5}\right)(m)(n) && \textit{Commutative Property} \\ &= \left(\frac{2}{5} \cdot \frac{3}{5}\right)(m \cdot n) && \textit{Associative Property} \\ &= \frac{6}{25}mn && \textit{Simplify.} \end{aligned}$$

Your Turn

g. $2x(-4.5y)$

h. $\frac{5}{7}g\left(\frac{2}{3}h\right)$

When you add any two rational numbers, is the sum always a rational number? What happens when you subtract or multiply rational numbers? In all three cases, the result is a rational number.

Closure Property of Rational Numbers

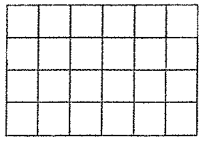
The sum, difference, or product of two rational numbers is always a rational number. Therefore, the set of rational numbers is closed under addition, subtraction, and multiplication.

For example, you can add, subtract, or multiply $\frac{5}{7}$ and $\frac{1}{7}$, and the result will be a rational number. *Check this.*

Check for Understanding

Communicating Mathematics

1. **State** whether $1.2bc$ is *sometimes*, *always*, or *never* a rational number if b and c are rational numbers. Explain.
2. **Determine** whether the product of three negative rational numbers can ever be positive. Explain.
3. **Write** a multiplication equation that can be shown using the model.



Exercise 3

Guided Practice

Getting Ready Find each product.

Sample: $(-4)(-6)$ **Solution:** $(-4)(-6) = 24$

4. $-1 \cdot 8$
5. $3(-3)$
6. $-7 \cdot (-4)$

Examples 1–6 Find each product.

7. $2.1(-7)$
8. $(-1.8)(-3.5)$
9. $0.5(-4.6)$
10. $-\frac{1}{3}\left(-\frac{4}{7}\right)$
11. $-3 \cdot \frac{3}{4}$
12. $2\frac{3}{5}\left(\frac{1}{4}\right)$

Example 7 Simplify each expression.

13. $3\left(\frac{1}{5}y\right)$
14. $-\frac{1}{4} \cdot \frac{7}{10}k$
15. $-2.5c(-2.4d)$

Example 5

16. **Health** Students who carry backpacks that are too heavy can develop spinal problems. The maximum recommended backpack weight y is given by the equation $y = \frac{3}{20}x$, where x is a student's weight. Would a 12-pound backpack be too heavy for a student who weighs 110 pounds? Explain.
Source: *Zillions*

Exercises

Practice

Homework Help

For Exercises	See Examples
17–25, 48	1, 2
26–34	4, 5, 6
35–42	3, 7, 8
43, 45–47	3

Extra Practice
See page 698.

Find each product.

17. $4 \cdot 6.1$
18. $3.8(8)$
19. $7.1(-1)$
20. $(-0.5)(-5.6)$
21. $-6.0(8.5)$
22. $-0.4(1.5)$
23. $-2.9 \cdot 10$
24. $-7.3 \cdot 0$
25. $(-12.8)(-1)(4.5)$
26. $\frac{1}{5}\left(\frac{2}{3}\right)$
27. $-1 \cdot \left(-\frac{5}{8}\right)$
28. $-\frac{4}{7} \cdot \frac{3}{5}$
29. $(0)\left(-\frac{6}{7}\right)$
30. $-\frac{3}{5}\left(-\frac{9}{10}\right)$
31. $\frac{7}{3}\left(-\frac{3}{7}\right)$
32. $\left(-\frac{3}{5}\right)\left(\frac{-2}{7}\right)(-1)$
33. $3 \cdot \frac{7}{8}$
34. $\frac{6}{7}\left(-2\frac{1}{3}\right)$

Simplify each expression.

35. $2(-1.5x)$
36. $0.6(-15y)$
37. $(4r)(2.2s)$
38. $\frac{5}{6}r(-18s)$
39. $\left(\frac{3}{5}a\right)\left(\frac{2}{5}b\right)$
40. $\frac{1}{3}y\left(-\frac{1}{4}z\right)$
41. $-\frac{1}{2}s\left(-\frac{1}{2}\right) + \frac{2}{3}\left(\frac{3}{2}s\right)$
42. $\frac{5}{6}\left(\frac{3}{4}a\right) - \left(\frac{1}{8}a\right)\left(-\frac{1}{3}\right)$

43. Solve $x = 2(-3.2) - 0.5(-4.8)$ to find the value of x.

44. Evaluate $7a(-4.5b) + (-2ab)$ if $a = 2$ and $b = -1$.

Applications and Problem Solving

45. **Shopping** Teri wants to buy a bicycle with the money she earns from her after-school job. If she can save $22.50 a week, how much money will she have after six weeks?

46. **Space** A day on Jupiter is much shorter than a day on Earth because Jupiter rotates on its axis about $2\frac{2}{5}$ times as fast as Earth rotates on its axis. During our month of June, how many days will Jupiter have?

47. **History** Galileo (1564–1642) was a mathematician and scientist who is said to have experimented with falling objects from the Leaning Tower of Pisa. Suppose Galileo dropped a stone from the tower and it took $3\frac{1}{2}$ seconds to reach the ground. Solve $d = -\frac{1}{2}\left(9\frac{4}{5}\right)\left(3\frac{1}{2}\right)\left(3\frac{1}{2}\right)$ to find the distance d in meters that the stone fell. *A negative answer means that the final position is lower than the initial position.*

48. **Critical Thinking** Describe the values of a and b for each statement to be true.

a. ab is positive. b. ab is negative. c. $ab = 0$

Mixed Review

Solve each equation. Check your solution. *(Lesson 3–7)*

49. $|t| = -5$ 50. $2 + |x| = 6$ 51. $3 = |-9 - a|$

52. **Aviation** A traffic helicopter descended 160 meters to observe road conditions. It leveled off at 225 meters. *(Lesson 3–6)*

a. Let a represent the original altitude. Write an equation to represent this problem.

b. What was the helicopter's original altitude?

Find each sum or difference. *(Lesson 3–2)*

53. $3.5 + (-2.1)$ 54. $-1.7 - 4.0$ 55. $-\frac{3}{4} + \frac{1}{8}$

56. Find $24 \div (-6)$. *(Lesson 2–6)*

Standardized Test Practice Ⓐ Ⓑ Ⓒ Ⓓ

57. **Extended Response** The stem-and-leaf plot shows the heights of 30 students. *(Lesson 1–7)*

a. What is the height of the tallest student?

b. Which height occurs most frequently?

Class Heights

Stem	Leaf
5	7 8 8 9 9 9
6	0 0 1 2 2 2 3 3 4 4 4 4 4
	5 5 7 8 9 9
7	0 0 2 3 3

7|2 = 72 in.

58. **Multiple Choice** The expression $(220 - y) \times 0.8 \div 4$ gives the target 15-second heart rate for an athlete y years old during a workout. Find the target 15-second heart rate for a 17-year-old athlete during a workout. *(Lesson 1–2)*

Ⓐ 27 Ⓑ 40.6 Ⓒ 216.6 Ⓓ 219.7