

4-2

Counting Outcomes

What You'll Learn

You'll learn to use tree diagrams and the Fundamental Counting Principle to count outcomes.

Why It's Important

Manufacturing Car manufacturers count outcomes to determine the number of different key combinations that are possible. See Exercise 23.

Baroness Martine de Beausoleil was a French scientist in the 17th century who spent 30 years studying geology and mathematics. She determined which rocks were valuable by the minerals they contained. Some other ways to classify rocks are by their texture and by their color.

How many different rocks are possible having the characteristics shown in the table? We can represent this situation by using a **tree diagram**.

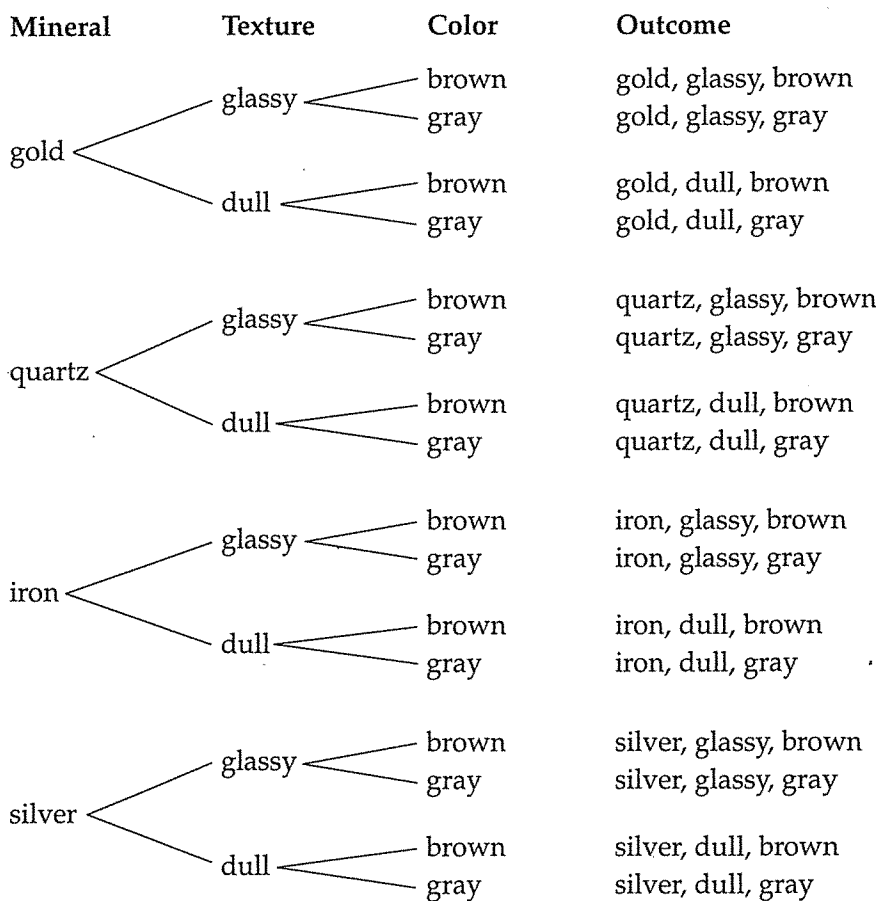
Rock Characteristics		
Mineral	Texture	Color
gold	glassy	brown
quartz	dull	gray
iron		
silver		



Quartz



Gold



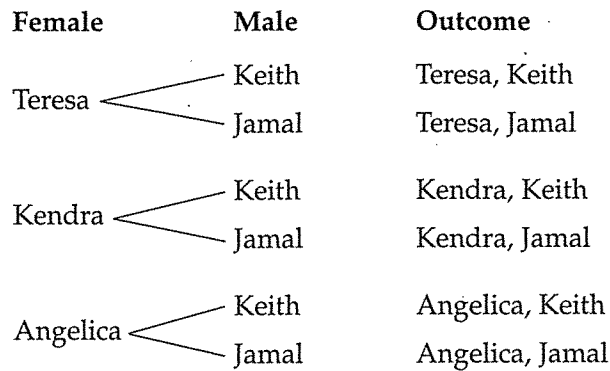
By using the tree diagram, we find that there are 16 different rocks possible. These results are called **outcomes**. For example, the first outcome is a rock that contains gold, has a glassy texture, and is brown. The list of all the possible outcomes is called the **sample space**.

Example
Dance Link

1

The Greenwood Dance Company has three lead female dancers, Teresa, Kendra, and Angelica, and two lead male dancers, Keith and Jamal. How many different ways can the director choose one female dancer and one male dancer to lead the next production?

Make a tree diagram to find the number of combinations.



The sample space contains six outcomes. So, there are six different ways that the director can choose the leads for the next production.

Your Turn

- a. The Ice Cream Parlor offers the choices below for making sundaes. Draw a tree diagram to find the number of different sundaes that can be made.

Ice Cream	Topping	Whipped Cream
chocolate	chocolate	yes
vanilla	butterscotch	no
rocky road		



An **event** is a subset of the possible outcomes, or sample space. In Example 1, the choice of female dancers is one event, and the choice of male dancers is another event. Notice that the product of the number of choices in each event is $3 \cdot 2$ or 6. This method of finding the number of possible outcomes is called the **Fundamental Counting Principle**.

Fundamental Counting Principle

If event M can occur in m ways and is followed by event N that can occur in n ways, then the event M followed by event N can occur in $m \times n$ ways.



Example
Clothing Link

2

How many different kinds of school sweatshirts are possible?

There are 3 styles, 2 colors, and 5 sizes, so the number of different sweatshirts is $3 \times 2 \times 5$ or 30.

Style	Color	Size
school name	red	small
school logo	tan	medium
team graphic		large
		1X
		2X

Your Turn

b. Catina wants to buy a red sweatshirt in size large or 1X in any style. How many choices does she have?

Check for Understanding

Communicating Mathematics

1. Explain the Fundamental Counting Principle in your own words.
2. Represent a real-life situation by drawing a tree diagram. Include a description of each event and a list of all the outcomes in the sample space.
3. **YOU?** Ling says that if three coins are tossed, then the number of outcomes with 2 heads and 1 tail is the same as the number with 1 head and 2 tails. Lorena thinks he is wrong. Who is correct and why?

Vocabulary

- tree diagram
- outcome
- sample space
- event

Guided Practice

Getting Ready

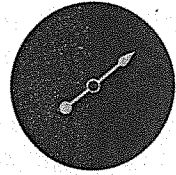
Suppose you spin the spinner twice. Determine whether each is an *outcome* or a *sample space*.

Sample 1: (red, blue)

Solution: Spinning a red and then a blue (red, blue) is an outcome because it is only one possible result.

Sample 2: (red, red), (red, blue), (blue, red), (blue, blue)

Solution: This is a sample space because it is a list of all possible outcomes.



Determine whether each is an *outcome* or a *sample space* for the given experiment.

4. (5, 2, 2); rolling a number cube three times
5. (H, T); tossing a coin once
6. (H, H), (H, T), (T, H), (T, T); tossing a coin twice
7. (4, 10, J, 3, 7); choosing five cards from a standard deck

Example 1

8. Suppose you can order a burrito or a taco with beef, chicken, or bean filling. Find the number of possible outcomes by drawing a tree diagram.

Example 2

9. Suppose you roll a die twice. Find the number of possible outcomes by using the Fundamental Counting Principle.

Example 1

10. **Shopping** Enrique wants to buy a bicycle, but he is having trouble deciding what kind to buy.

Type	Speed	Handlebars
road	21	touring
mountain	16	racing
	14	
	10	

- Draw a tree diagram to represent his choices.
- How many different kinds of bicycles are possible?
- How many different kinds of bicycles have racing handlebars and 16 or greater gear speeds?

Exercises

Practice

Find the number of possible outcomes by drawing a tree diagram.

- three tosses of a coin
- choosing one marble from each box shown in the table at the right

Box A	Box B	Box C
blue	green	green
green	red	white

- Anita purchased five T-shirts and three pairs of jeans for school, as shown in the table at the right. How many different T-shirt and jeans outfits are possible?

T-Shirt	Jeans
red	black
orange	white
green	blue
white	
striped	

Homework Help	
For Exercises	See Examples
11, 13, 15, 17, 19–22	1
12, 14, 16, 18	2
23	1, 2
Extra Practice	
See page 699.	

Find the number of possible outcomes by using the Fundamental Counting Principle.

- different cars with options shown at the right
- possible sequences of answers on a 5-question true-false quiz

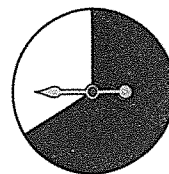
Color	Engine	Transmission
red	4-cylinder	manual
blue	6-cylinder	automatic
white		
green		

- different 1-topping pizzas with the choices shown at the right

Crust	Size	Topping	
thin	individual	pepperoni	onion
regular	small	mushroom	olive
thick	medium	sausage	pepper
	large		

17. Refer to Example 1. How many different ways can the director choose one female dancer and one male dancer if there are five female dancers and six male dancers?

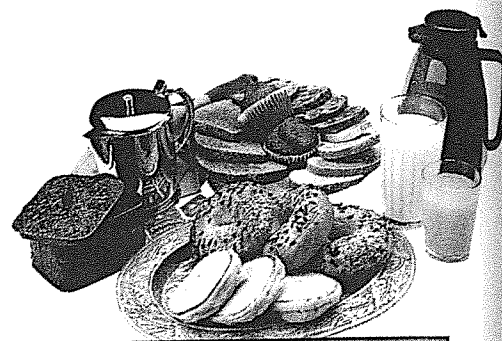
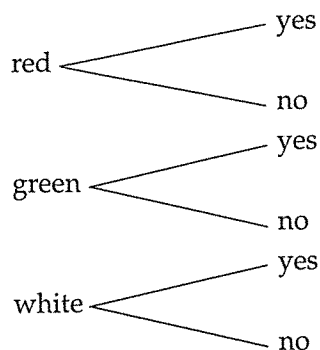
18. Suppose the spinner at the right is spun twice. Assume that the spinner will not land on a border.



a. Let R represent red, B represent blue, and Y represent yellow. Write all possible outcomes in the sample space as ordered pairs.

b. How many outcomes have at least one blue?

19. Write a situation that fits the tree diagram below.



Applications and Problem Solving

20. **Dining** A free continental breakfast offers one type of bread and one beverage.

a. How many different breakfasts are possible?

b. How many of the breakfasts include a muffin?

Bread	Beverage
bagel	tea
bran muffin	coffee
English muffin	juice
white toast	milk
wheat toast	
raisin toast	

21. **History** In 1869, Fanny Jackson Coppin became the first African-American school principal. One of her favorite teaching poems began:

A noun is the name of anything,
 As school, or garden, hoop, or swing.
 Adjectives tell the kind of noun,
 As great, small, pretty, white, or brown.

How many two-word phrases can you make using one of the underlined nouns and one of the underlined adjectives from the poem?

22. **Family** Valerie and Jessie just got married. They hope to eventually have two girls and a boy, in any order.

a. How many combinations of three children are possible?

b. How many outcomes will give them two girls and a boy?

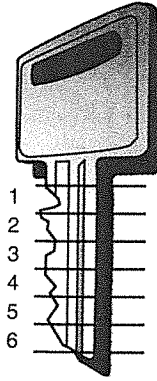
c. In how many of the outcomes from part b will the girls be born in consecutive order?

internet
CONNECTION

Data Update For the latest information on the number of key combinations, visit: www.algconcepts.com

23. **Keys** A car manufacturer makes keys with six sections.

- Until the 1960s, there were only two patterns for each section. How many different keys were possible?
- After the 1960s, the car manufacturer made keys having three different patterns for each section. How many different keys were possible after the 1960s?



24. **Critical Thinking** The president, vice president, secretary, and treasurer of the Drama Club pose for a yearbook picture. If they sit in four chairs, how many different seating arrangements are possible?

Mixed Review

Find each product. (Lesson 4-1)

25. $-3 \cdot 5.4$

26. $-7.2(-1.5)$

27. $\frac{3}{5} \cdot \frac{2}{7}$

28. **Manufacturing** A certain bolt used in lawn mowers will work properly only if its diameter differs from 2 centimeters by exactly 0.04 centimeter. (Lesson 3-7)

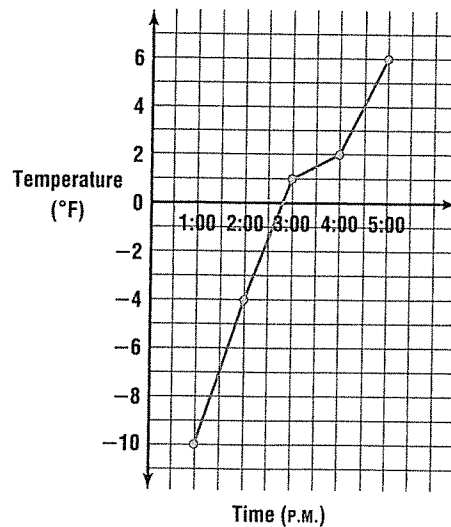
- Let d represent the diameter. Write an equation to represent this problem.
- What are the least and greatest diameters for this bolt?

29. Solve $x - \frac{1}{3} = -\frac{3}{4}$. Check your solution. (Lesson 3-6)

30. **Grid In** In the graph below, how many degrees did the temperature rise from 1:00 to 5:00? (Lesson 2-4)

Standardized Test Practice

A B C D



31. **Multiple Choice** Ayani has \$600 in the bank at an annual interest rate of 3%. How much money will he have in his account after four years? (Hint: Use the formula for simple interest $I = prt$.) (Lesson 1-5)

A \$72

B \$672

C \$607

D \$720

