4–3 Dividing Rational Numbers

What You'll Learn
You'll learn to divide rational numbers.

Why It's Important
Cooking To adjust recipes for different serving sizes, cooks must know how to divide rational numbers.
See Example 5.

When you divide two rational numbers, the quotient is also a rational number. You can use the rules below to find the sign of the quotient. The sign rules are the same as the ones used for dividing integers.

Dividing Rational Numbers	Words	Numbers
Different Signs	The quotient of two numbers with different signs is negative.	$-2.1 \div 7 = -0.3$ $2.1 \div (-7) = -0.3$
Same Sign	The quotient of two numbers with the same sign is positive.	$4.5 \div 0.9 = 5$ $-4.5 \div (-0.9) = 5$

Examples

Find each quotient.

1. $-6 \div 0.5$

$-6 \div 0.5 = -12$ *Numbers have different signs. The quotient is negative.*

2. $-7.4 \div (-2)$

$-7.4 \div (-2) = 3.7$ *Numbers have the same sign. The quotient is positive.*

Your Turn

a. $16 \div (-2.5)$ b. $-3.9 \div 3$ c. $-8.4 \div (-1.2)$

Prerequisite Skills Review
Operations with Decimals, p. 684

Two numbers whose product is 1 are called **multiplicative inverses** or **reciprocals**.

$\frac{7}{8}$ and $\frac{8}{7}$ are reciprocals because $\frac{7}{8} \cdot \frac{8}{7} = 1$.

-3 and $-\frac{1}{3}$ are reciprocals because $-3 \cdot \left(-\frac{1}{3}\right) = 1$.

a and $\frac{1}{a}$, where $a \neq 0$, are reciprocals because $a \cdot \frac{1}{a} = 1$.

These examples demonstrate the **Multiplicative Inverse Property**.

Multiplicative Inverse Property		
	Words:	The product of a number and its multiplicative inverse is 1.
	Symbols:	For every number $\frac{a}{b}$, where $a, b \neq 0$, there is exactly one number $\frac{b}{a}$ such that $\frac{a}{b} \cdot \frac{b}{a} = 1$.
	Numbers:	$-\frac{4}{9}$ and $-\frac{9}{4}$ are multiplicative inverses.

In Lesson 2–4, you learned that subtracting a number is the same as adding its additive inverse. In the same way, dividing by a number is the same as multiplying by its multiplicative inverse or reciprocal.

Dividing Fractions

Words: To divide a fraction by any nonzero number, multiply by the reciprocal of the number.

Symbols: $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$, where $b, c, d \neq 0$

Numbers: $\frac{3}{4} \div \frac{2}{5} = \frac{3}{4} \cdot \frac{5}{2}$ or $\frac{15}{8}$

Use the rules for dividing rational numbers to determine the sign of the quotient.

Examples

Find each quotient.

3 $\frac{4}{7} \div (-8)$

$\frac{4}{7} \div (-8) = \frac{4}{7} \cdot \left(-\frac{1}{8}\right)$ *To divide by -8, multiply by its reciprocal, $-\frac{1}{8}$.*

$= -\frac{4}{56}$ *The numbers have different signs. The product is negative.*

$= -\frac{1}{14}$ *Write the fraction in simplest form.*

Prerequisite Skills Review
Simplifying Fractions, p. 685

4 $\frac{5}{6} \div \left(1\frac{2}{5}\right)$

$\frac{5}{6} \div \left(1\frac{2}{5}\right) = \frac{5}{6} \div \frac{7}{5}$ *Rewrite $1\frac{2}{5}$ as an improper fraction.*

$= \frac{5}{6} \cdot \frac{5}{7}$ *To divide by $\frac{7}{5}$, multiply by its reciprocal, $\frac{5}{7}$.*

$= \frac{25}{42}$ *The numbers have the same sign. The product is positive.*

Real World **Example** **5** **Cooking Link**

How much sugar do you need to make one dozen cookies?

Divide the recipe by 2. The recipe calls for $\frac{2}{3}$ cup of sugar.

$\left(\begin{matrix}\text{amount} \\ \text{of sugar}\end{matrix}\right) \div 2 = \frac{2}{3} \div 2$

$= \frac{2}{3} \cdot \frac{1}{2}$

$= \frac{2}{6}$ or $\frac{1}{3}$ *Write the fraction in simplest form.*

To make one dozen cookies, you need $\frac{1}{3}$ cup of sugar.

Raspberry Almond Cookies

1 cup butter
2 cups flour
$\frac{2}{3}$ cup sugar
$\frac{1}{2}$ cup raspberry jam
$\frac{1}{2}$ tsp. almond extract
Makes two dozen cookies.

Your Turn Find each quotient.

d. $14 \div \left(-\frac{2}{3}\right)$ e. $-\frac{2}{7} \div \left(-\frac{5}{9}\right)$ f. $\frac{1}{6} \div 2\frac{4}{5}$

You can use what you know about dividing rational numbers to evaluate algebraic expressions.

Example 6 Evaluate $\frac{x}{4}$ if $x = \frac{3}{5}$.

$\frac{x}{4} = \frac{\frac{3}{5}}{4}$ *Replace x with $\frac{3}{5}$.*

$= \frac{3}{5} \div 4$ *Rewrite the fraction as a division sentence.*

$= \frac{3}{5} \cdot \frac{1}{4}$ *To divide by 4, multiply by its reciprocal, $\frac{1}{4}$.*

$= \frac{3}{20}$ *Multiply the numerators and multiply the denominators.*

Your Turn Evaluate if $x = \frac{3}{5}$.

g. $\frac{6}{x}$ h. $-\frac{x}{2}$ i. $\frac{4}{7x}$

Check for Understanding

Communicating Mathematics

1. **Determine** whether the reciprocal of a negative rational number can ever be positive. Explain.
2. **Compare** a positive rational number n with the value of $n \div \frac{1}{2}$ and with the value of $n \cdot \frac{1}{2}$. Which is the greatest? the least?
3. Writing Math Describe real-life situations in which you would divide fractions to solve problems.

Vocabulary
multiplicative inverse
reciprocal

Guided Practice

Getting Ready **Name the reciprocal of each number.**

Sample: $-\frac{1}{4}$ **Solution:** $-\frac{1}{4}\left(-\frac{4}{1}\right) = 1$, so the reciprocal is $-\frac{4}{1}$ or -4.

4. 3 5. $-\frac{2}{3}$ 6. $2\frac{3}{4}$ 7. $-\frac{1}{x}, x \neq 0$

Examples 1–5 **Find each quotient.**

8. $-6 \div 1.5$
9. $3.8 \div 19$
10. $-4.7 \div (-0.5)$
11. $-\frac{1}{2} \div \left(\frac{7}{3}\right)$
12. $4 \div \left(\frac{7}{8}\right)$
13. $2\frac{2}{5} \div \frac{1}{3}$

Example 6 **Evaluate each expression if $a = \frac{1}{4}$ and $b = -\frac{2}{3}$.**

14. $\frac{a}{3}$
15. $\frac{5}{b}$
16. $\frac{a}{b}$

Example 4

17. **Lunch** Students working on decorations for homecoming ordered pizza for lunch. They ordered five 8-slice pizzas, and each person ate $2\frac{1}{2}$ slices. There was no pizza left over. How many students were there?

Exercises

Practice

Find each quotient.

18. $8 \div (-1.6)$
19. $-7.5 \div 3$
20. $9.6 \div 3.2$
21. $2.7 \div -27$
22. $0.4 \div -0.4$
23. $15.6 \div 1.3$
24. $\frac{1}{8} \div \frac{1}{7}$
25. $0 \div \frac{3}{8}$
26. $\frac{3}{4} \div \left(-\frac{4}{5}\right)$
27. $-\frac{5}{6} \div \frac{5}{6}$
28. $\frac{1}{-7} \div \frac{4}{9}$
29. $-\frac{2}{3} \div \left(-\frac{5}{7}\right)$
30. $-\frac{7}{9} \div \left(-\frac{9}{7}\right)$
31. $5 \div \frac{1}{9}$
32. $-\frac{1}{5} \div 10$
33. $\frac{-3}{4} \div 8$
34. $-14 \div \left(-2\frac{4}{5}\right)$
35. $-3\frac{1}{2} \div \frac{1}{6}$

Homework Help

For Exercises	See Examples
18–23	1, 2
24–35	3, 4
36–49	5, 6

Extra Practice

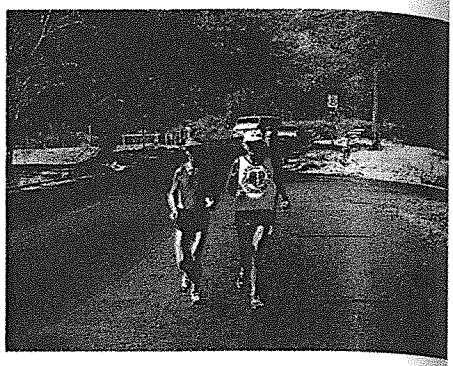
See page 699.

Evaluate each expression if $j = -\frac{1}{2}$, $k = \frac{3}{4}$, and $n = \frac{1}{5}$.

36. $\frac{3}{n}$
37. $\frac{9}{k}$
38. $\frac{k}{2}$
39. $\frac{-j}{5}$
40. $\frac{1}{2n}$
41. $\frac{n}{j}$
42. $\frac{k}{j}$
43. $\frac{2n}{j}$
44. $\frac{jk}{n}$

45. Find the quotient of -12 and $-\frac{2}{3}$.

46. Solve the equation $t = -\frac{7}{18} \div 3\frac{1}{2}$ to find the value of t.



Applications and Problem Solving

47. **Track** Mr. Vance is training for a 10-kilometer race in a senior citizen track meet. Suppose $3\frac{1}{4}$ laps equals 1 kilometer and he wants to finish the race in 65 minutes. In how many minutes will he have to run each lap?

48. **Nutrition** A 20-ounce bottle of soda contains $16\frac{2}{3}$ teaspoons of sugar. A 12-ounce can of soda contains $16\frac{2}{3} \div \frac{5}{3}$ teaspoons of sugar. How much sugar is in a 12-ounce can of soda?

49. **Sales** The performing arts series at Central State University offers a package ticket to students for \$62.50. If there are five performances, how much do students pay for each performance?

50. **Critical Thinking**
   a. Is the division of two rational numbers always a rational number? If not, give a counterexample.
   b. Is the set of rational numbers closed under division? Explain.
   c. What number has no reciprocal? Explain.

Mixed Review

51. **Uniforms** The school dress for female students is a uniform, a blouse, and socks. A sweater may be added, if necessary. Find the number of different outfits that female students can wear. *(Lesson 4–2)*

Uniform	Blouse	Socks	Sweater
skirt jumper	white yellow blue pink	white blue gray	yes no

52. **Sales** Ms. Ortiz wants to buy a washer and dryer from Bargain Appliances. If she buys on the "90 days same as cash" plan, she will have to make 5 equal payments of \$146.12 each. If she writes a check for each payment, what is the net effect on her checking account? *(Lesson 4–1)*

Find the mean, median, mode, and range for each set of data. *(Lesson 3–3)*

53. 6, 8, 8, 8, 6, 4, 3, 7, 4

54. 9, 4, 13, 11, 7, 6, 20

55. 20, 11, 20, 23, 20, 29

Replace each ● with <, >, or = to make a true sentence. *(Lesson 3–1)*

56. $-3.6 \bullet 0$

57. $\frac{3}{10} \bullet 0.3$

58. $-7(-4) \bullet 7 - 15$

Which is the better buy? Explain. *(Lesson 3–1)*

59. a $\frac{1}{2}$-pound bag of cashews for \$3.15 or $\frac{3}{4}$-pound bag for \$5.19

60. three liters of soda for \$2.25 or two liters for \$1.98

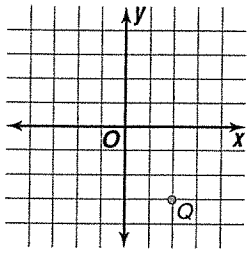
61. a 48-ounce bottle of dishwashing liquid for \$2.69 or a 22-ounce bottle for \$1.09

Standardized Test Practice

Ⓐ Ⓑ Ⓒ Ⓓ

62. **Grid In** Evaluate $16 + k$ if $k = -11$. *(Lesson 2–3)*

63. **Multiple Choice** Write the ordered pair that names point Q. *(Lesson 2–2)*



Ⓐ (2, −3) Ⓑ (−2, −3)

Ⓒ (−3, 2) Ⓓ (−3, −2)